

I will be able to discover and explain the sum of the measures of a triangle, develop inductive and deductive reasoning, and practice using geometry tools.

I will do this via capturing notes in Cornell notes format, whole class discussions to introduce and clarify key concepts, and completing practice problems collaboratively with teacher support.

#### 4.1 Triangle Sum Conjecture



This is a partner activity.

Each pair needs:

- 1 protractor
- 1 yellow half-sheet of paper
- 1 writing instrument
- 1 pair of scissors

- Draw three different triangles on one side of the paper.
- Each partner carefully measures the three angles of each triangle and finds the sum of the angles of each triangle.
- Compare your sums with your partner. If they are not the same, come up with some reasons why they might differ.
- Carefully cut out the triangles and label the three corners of each.
- Carefully tear one of the three triangles into three sections each so that one corner from the triangle is on each piece.
- Arrange the pieces of one triangle so that the vertices meet.
- Continue with the other two triangles.
- Propose a conjecture for your results.

In mathematics, a **conjecture** is a mathematical statement which appears to be true, but has not been formally proven. A **conjecture** can be thought of as the mathematicians way of saying “I believe that this is true, but I have no proof yet”. A **conjecture** is a good guess or an idea about a pattern.

## 4.1 Triangle Sum Conjecture

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

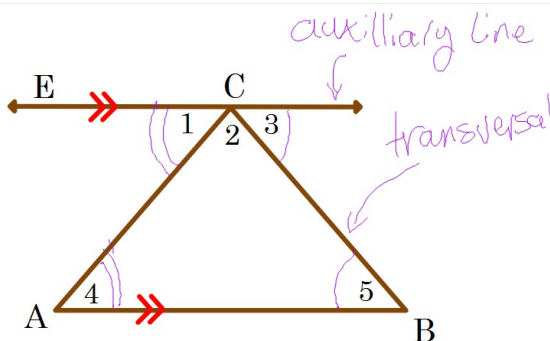
straight line

$$\angle 1 \cong \angle 4, \text{ transversal AIA}$$

$$\angle 3 \cong \angle 5, \text{ " "}$$

$$\angle 4 + \angle 2 + \angle 5 = 180^\circ$$

$\therefore$  the 3 angles of a  $\triangle \leq$  to  $180^\circ$



$\Sigma$  Sigma stands for Sum (+)

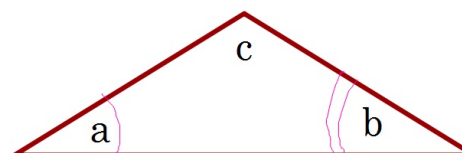
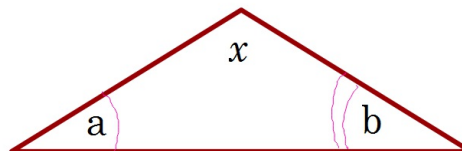
Given parallel lines EC and AB with transversals AC and BC, and angles 1, 2, and 3 are colinear. Angles 1, 2, and 3 will sum to 180 degrees. Angles 1 and 4 are congruent (AIA). Angles 3 and 5 are congruent (AIA). Substituting congruent angles into the sum, angles 4, 2, and 5 will sum to 180 degrees. Therefore, the three angles of a triangle sum to 180 degrees.

I will be able to discover and explain the sum of the measures of a triangle, develop inductive and deductive reasoning, and practice using geometry tools.

## 4.1 Triangle Sum Conjecture

### Third Angle Conjecture

If two triangles have two congruent angles then the third angle must also be congruent.



$$\angle a + \angle b + \angle x = 180^\circ$$

$$\angle a + \angle b + \angle c = 180^\circ$$

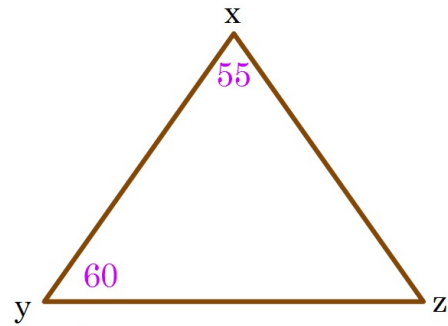
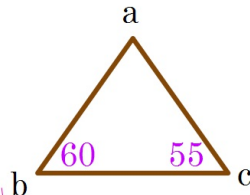
$$\therefore \angle c \cong \angle x$$

I will be able to discover and explain the sum of the measures of a triangle, develop inductive and deductive reasoning, and practice using geometry tools.

### 4.1 Triangle Sum Conjecture

$$\begin{cases} \angle b \cong \angle y \\ \angle c \cong \angle x \\ \therefore \angle a \cong \angle z \end{cases}$$

Given two triangles  
With two congruent  
angles, by the third angle  
conjecture the remaining  
angles must be congruent.



Similar  
triangles -  
congruent angles, but different  
side lengths

I will be able to discover and explain the sum of the measures of a triangle, develop inductive and deductive reasoning, and practice using geometry tools.

### 4.1 Triangle Sum Conjecture

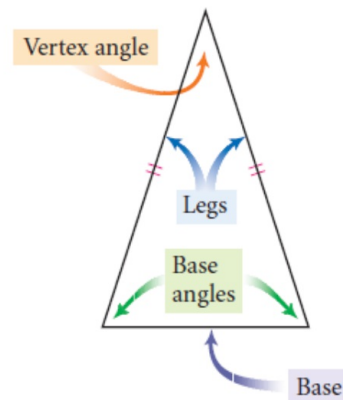
Exercises DG pp. 201-202 #2-22 Evens & #9  
With a partner

I will be able to discover and explain the sum of the measures of a triangle, develop inductive and deductive reasoning, and practice using geometry tools.

IWBAT discover a relationship between the base angles of an isosceles triangle.

I will do this via capturing notes in Cornell notes format, whole class discussions to introduce and clarify key concepts, and completing practice problems collaboratively with teacher support.

## 4.2 Properties of Special Triangles



Complete Investigation 1 on p. 205 and complete the Isosceles Triangle Conjecture.

Isosceles Triangle Conjecture

If a triangle is isosceles, then the base angles are congruent.

IWBAT discover a relationship between the base angles of an isosceles triangle.



## 4.2 Properties of Special Triangles

Is the converse true?

Perform the Investigation on p. 206 and complete the converse below.

Converse of the Isosceles Triangle Conjecture

If a triangle has two congruent angles, then

it's an isosceles.

IWBAT discover a relationship between the base angles of an isosceles triangle.

## 4.2 Properties of Special Triangles

$$\frac{180-22}{2} = m\angle H$$

Exercises DG pp. 206-207 #1-8, 10, 11

IWBAT discover a relationship between the base angles of an isosceles triangle.

IWBAT review the relationship between the constant term in a linear equation in the form of  $y = mx + b$  and the y-intercept of an equation's graph and review the relationship between the coefficient of x and the slope of a graph of an equation in slope-intercept form.

I will do this via capturing notes in Cornell notes format, whole class discussions to introduce and clarify key concepts, and completing practice problems collaboratively with teacher support.

$$f = \frac{9}{5}c + 32$$

$y = mx + b$   
y-intercept form  
slope-intercept

$$f - 212 = \frac{9}{5}(c - 100)$$

point-slope form  
 $y - y_1 = m(x - x_1)$   
 $y = m(x - x_1) + y_1$   
point  $(x_1, y_1)$

$$c = \frac{5}{9}(f - 32)$$

$$f = \frac{9}{5}(c - 100) + 212$$

IWBAT review the relationship between the constant term in a linear equation in the form of  $y = mx + b$  and the y-intercept of an equation's graph and review the relationship between the coefficient of x and the slope of a graph of an equation in slope-intercept form.

## Writing Linear Equations

Find the equation of  $\overline{AB}$  from its graph.

$$y = mx + b$$

intercept  $(0, -2)$   $b = -2$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{3}{4}$$

$$y = \frac{3}{4}x - 2$$

$$y - y_1 = m(x - x_1)$$

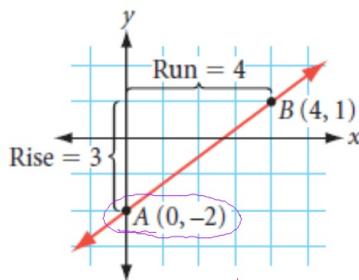
$$y - 1 = \frac{3}{4}(x - 4)$$

$$y + 2 = \frac{3}{4}x$$

$$y - 1 = \frac{3}{4}x - 3$$

$$+3 \quad +3$$

$$y + 2 = \frac{3}{4}x$$



correct

IWBAT review the relationship between the constant term in a linear equation in the form of  $y = mx + b$  and the y-intercept of an equation's graph and review the relationship between the coefficient of  $x$  and the slope of a graph of an equation in slope-intercept form.

## Writing Linear Equations

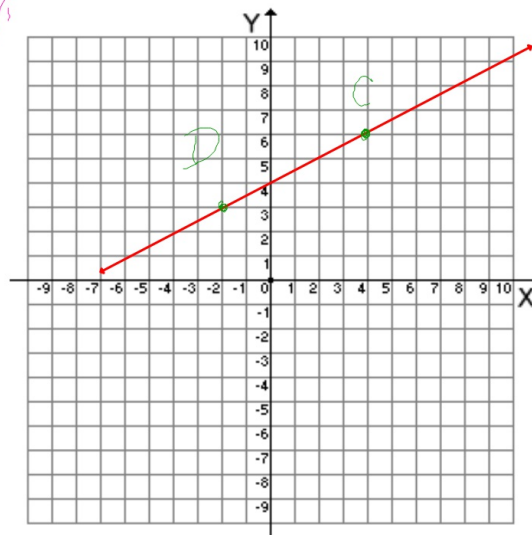
Given points  $C(4, 6)$  and  $D(-2, 3)$ , find the equation of  $\overline{CD}$ .

$$m = \frac{3}{6} = \frac{1}{2}$$

$$y = mx + b$$

$$y = m(x - x_1) + y_1$$

$$y = \frac{1}{2}x + 4$$



IWBAT review the relationship between the constant term in a linear equation in the form of  $y = mx + b$  and the y-intercept of an equation's graph and review the relationship between the coefficient of  $x$  and the slope of a graph of an equation in slope-intercept form.

## Writing Linear Equations

Find the equation of the perpendicular bisector of the segment with endpoints (2, 9) and (-6, -7).

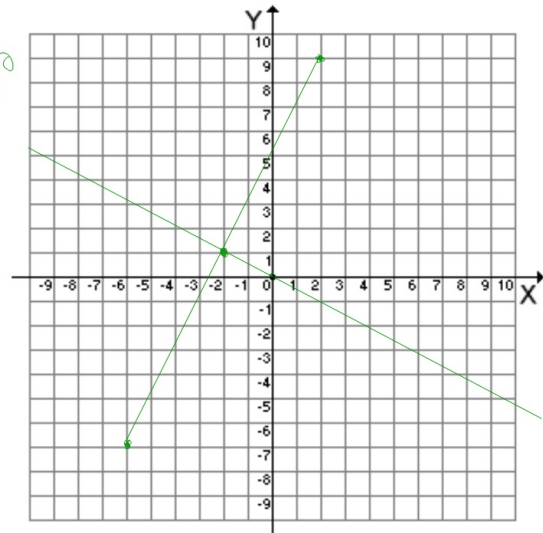
$$\left(\frac{2 + -6}{2}, \frac{9 + -7}{2}\right) = (-2, 1)$$

⊥ perpendicular, meets @ 90°

$$m = \frac{9 - -7}{2 - -6} = \frac{16}{8} = 2$$

$$m_{\perp} = \frac{-1}{m} = \frac{-1}{2}$$

$$y - 1 = -\frac{1}{2}(x + 2)$$



IWBAT review the relationship between the constant term in a linear equation in the form of  $y = mx + b$  and the y-intercept of an equation's graph and review the relationship between the coefficient of x and the slope of a graph of an equation in slope-intercept form.

## Writing Linear Equations

Exercises p.212 #4-8, 11

IWBAT review the relationship between the constant term in a linear equation in the form of  $y = mx + b$  and the y-intercept of an equation's graph and review the relationship between the coefficient of x and the slope of a graph of an equation in slope-intercept form.



### 4.3 Triangle Inequalities

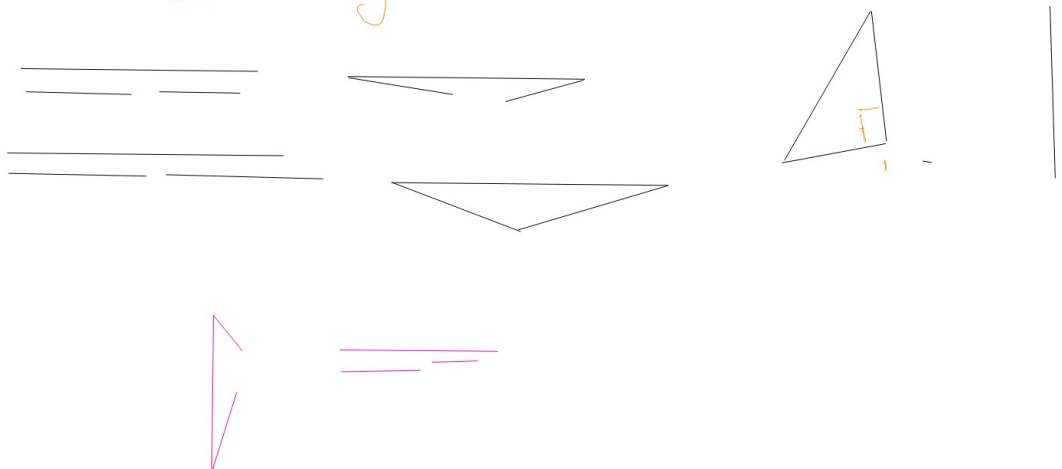
01/31/18

IWBAT investigate inequalities among sides and angles in triangles and discover the Exterior Angle Conjecture.

I will do this via capturing notes in Cornell notes format, whole class discussions to introduce and clarify key concepts, and completing practice problems collaboratively with teacher support.

### 4.3 Triangle Inequalities

Can you form a triangle using sticks of any three lengths?



IWBAT investigate inequalities among sides and angles in triangles and discover the Exterior Angle Conjecture.

### 4.3 Triangle Inequalities

Each person at your table should do each construction.  
Compare results at your table when you finish.

Investigation 1 p.214

Construct:  $\triangle CAT$

/ / NO  
~

Construct:  $\triangle FSH$

NO  
~

### Triangle Inequality Conjecture

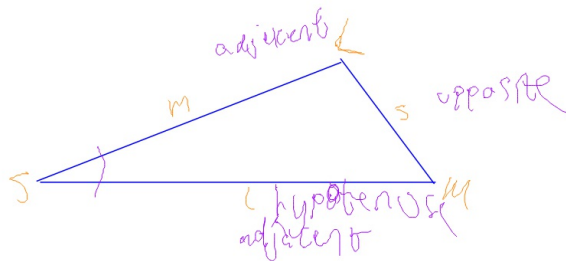
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

IWBAT investigate inequalities among sides and angles in triangles and discover the Exterior Angle Conjecture.

### 4.3 Triangle Inequalities

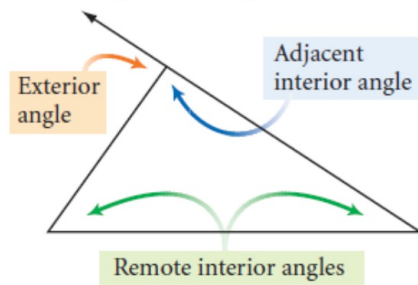
Each person should draw a different scalene triangle for this investigation. Some should draw acute triangles, and some should draw obtuse triangles.

1. Measure the angles in your triangle. Label the angle with greatest measure  $\angle L$ , the angle with second greatest measure  $\angle M$ , and the smallest angle  $\angle S$ .
2. Measure the three sides. Label the longest side  $l$ , the second longest side  $m$ , and the shortest side  $s$ .
3. Which side is opposite  $\angle L$ ?  $\angle M$ ?  $\angle S$ ?

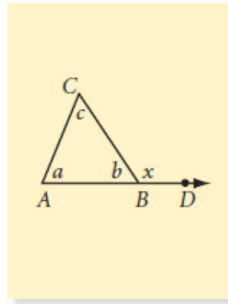


IWBAT investigate inequalities among sides and angles in triangles and discover the Exterior Angle Conjecture.

## 4.3 Triangle Inequalities



### Investigation 3 p. 215



### Triangle Exterior Angle Conjecture

The measure of an exterior angle of a triangle is the sum of the two remote interior angles.

IWBAT investigate inequalities among sides and angles in triangles and discover the Exterior Angle Conjecture.

## 4.3 Triangle Inequalities

Exercises pp.216-217 #2-16 evens

IWBAT investigate inequalities among sides and angles in triangles and discover the Exterior Angle Conjecture.

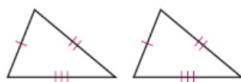
IWBAT explore shortcut methods for determining whether triangles are congruent.

I will do this via capturing notes in Cornell notes format, whole class discussions to introduce and clarify key concepts, and completing practice problems collaboratively with teacher support.

#### 4.4 Are there congruence shortcuts?

We will look at these congruence shortcuts to see if they are actually ones we can use to say that two triangles are congruent.

##### Side-Side-Side (SSS)



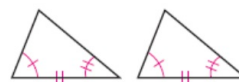
Three pairs of congruent sides

##### Side-Angle-Side (SAS)



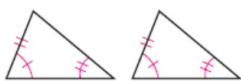
Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

##### Angle-Side-Angle (ASA)



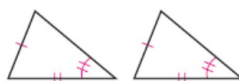
Two pairs of congruent angles and one pair of congruent sides (sides between the pairs of angles)

##### Side-Angle-Angle (SAA)



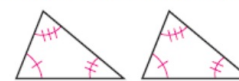
Two pairs of congruent angles and one pair of congruent sides (sides not between the pairs of angles)

##### Side-Side-Angle (SSA)



Two pairs of congruent sides and one pair of congruent angles (angles not between the pairs of sides)

##### Angle-Angle-Angle (AAA)



Three pairs of congruent angles

IWBAT explore shortcut methods for determining whether triangles are congruent.



#### 4.4 Are there congruence shortcuts?

Is **Side-Side-Side (SSS)** a congruence shortcut?



Three pairs of congruent sides

Everyone at your table shall construct their own triangle from the same three segments. Compare your triangles with each other. What do you notice?

4" 6" 3"

IWBAT explore shortcut methods for determining whether triangles are congruent.

#### 4.4 Are there congruence shortcuts?

**SSS Congruence Conjecture**

If the three sides of one triangle are congruent to the three sides of another triangle, then the triangles are congruent.

Is **Side-Angle-Side (SAS)** a congruence shortcut?



Two pairs of congruent sides and one pair of congruent angles (angles between the pairs of sides)

Everyone at your table shall construct their own triangle from the same two segments and included angle. Compare your triangles with each other. What do you notice?

5" 6" 20°

IWBAT explore shortcut methods for determining whether triangles are congruent.

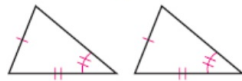
#### 4.4 Are there congruence shortcuts?

##### SAS Congruence Conjecture

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent

Is **Side-Side-Angle (SSA)**

a congruence shortcut?



Two pairs of congruent sides and one pair of congruent angles (angles not between the pairs of sides)

Everyone at your table shall construct their own triangle from the same two segments and non-included angle.

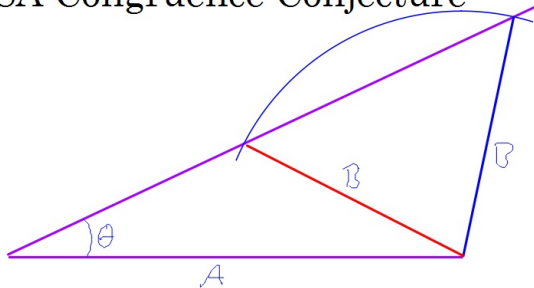
Compare your triangles with each other. What do you notice?

4" 2" 50°

IWBAT explore shortcut methods for determining whether triangles are congruent.

#### 4.4 Are there congruence shortcuts?

##### SSA Congruence Conjecture



Exercises pp.222-223 #1-5 odd, 9-15 odd

SSA can not be used to prove congruence

IWBAT explore shortcut methods for determining whether triangles are congruent.

#### 4.5 Are there other congruence shortcuts?

02/05/18

IWBAT explore shortcut methods for determining whether triangles are congruent.

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#### 4.5 Are there other congruence shortcuts?

Is **Angle-Angle-Angle (AAA)** a congruence shortcut?



Three pairs of congruent angles

Everyone at your table shall construct their own triangle from the same three angles. Compare your triangles with each other. What do you notice?

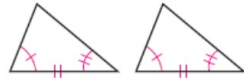
$20^\circ$   $60^\circ$   $100^\circ$

AAA Cannot be used to prove Congruency.

IWBAT explore shortcut methods for determining whether triangles are congruent.

#### 4.5 Are there other congruence shortcuts?

Is **Angle-Side-Angle (ASA)** a congruence shortcut?



Two pairs of congruent angles  
and one pair of congruent  
sides (sides between the pairs  
of angles)

Everyone at your table shall construct their own triangle from the same two angles and included segment. Compare your triangles with each other.

What do you notice?  $4''$   $40^\circ$   $80^\circ$



IWBAT explore shortcut methods for determining whether triangles are congruent.

#### 4.5 Are there other congruence shortcuts?

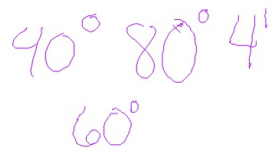
**ASA Congruence Conjecture**

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then The triangles are congruent

Is **Side-Angle-Angle (SAA)** a congruence shortcut?



Two pairs of congruent angles  
and one pair of congruent  
sides (sides not between the  
pairs of angles)



Everyone at your table shall construct their own triangle from the same two angles and non-included segment. Compare your triangles with each other. What do you notice?

IWBAT explore shortcut methods for determining whether triangles are congruent.



#### 4.5 Are there other congruence shortcuts?

##### SAA Congruence Conjecture

If two angles and a non-included side of one triangle are congruent to the corresponding angles and side of another triangle, then the triangles are similar but not congruent.

Exercises pp.227-228 #2-14 evens

IWBAT explore shortcut methods for determining whether triangles are congruent.

#### 4.6 Corresponding parts of congruent triangles

02/06/18

IWBAT show that pairs of angles or pairs of sides are congruent by identifying related triangles and proving them congruent, then applying CPCTC.

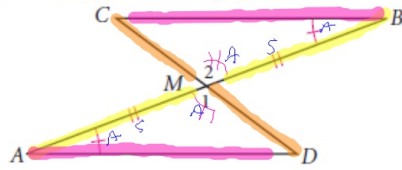
I will do this via capturing notes in Cornell notes format, whole class discussions to introduce and clarify key concepts, and completing practice problems collaboratively with teacher support.

## 4.6 Corresponding parts of congruent triangles

In the last two lessons, you discovered four shortcuts for showing that two triangles are congruent—SSS, SAS, ASA, and SAA. The definition of congruent triangles states that if two triangles are congruent, then the corresponding parts of those congruent triangles are congruent. We'll use the letters **CPCTC** to refer to the definition.

IWBAT show that pairs of angles or pairs of sides are congruent by identifying related triangles and proving them congruent, then applying CPCTC.

## 4.6 Corresponding parts of congruent triangles



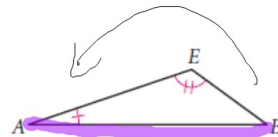
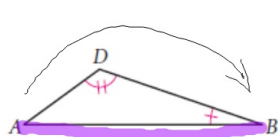
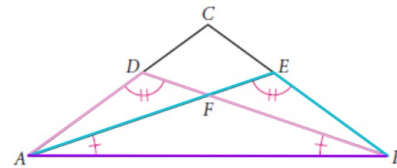
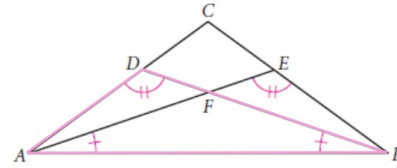
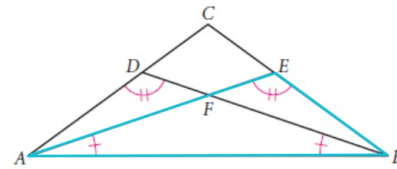
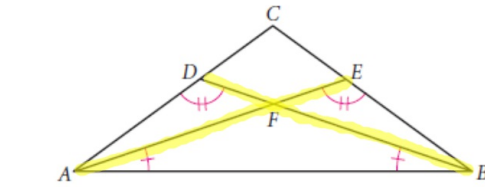
Is  $\overline{AD} \cong \overline{BC}$  in the figure above? Use a deductive argument to explain why they must be congruent.

|                                     |                 |
|-------------------------------------|-----------------|
| $\overline{AM} \cong \overline{BM}$ | Given           |
| $\angle A \cong \angle B$           | Given           |
| $\angle 1 \cong \angle 2$           | Vertical angles |
| $\triangle AMD \cong \triangle BMC$ | ASA             |
| $\overline{AD} \cong \overline{BC}$ | CPCTC           |

IWBAT show that pairs of angles or pairs of sides are congruent by identifying related triangles and proving them congruent, then applying CPCTC.

## 4.6 Corresponding parts of congruent triangles

Is  $\overline{AE} \cong \overline{BD}$ ? Write a paragraph proof explaining why.

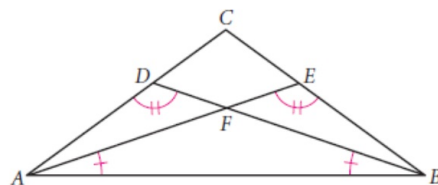


AAS

IWBAT show that pairs of angles or pairs of sides are congruent by identifying related triangles and proving them congruent, then applying CPCTC.

## 4.6 Corresponding parts of congruent triangles

Is  $\overline{AE} \cong \overline{BD}$ ? Write a paragraph proof explaining why.



Given that  $\angle ADB$  is congruent to  $\angle BEA$  and  $\angle EAB$  is congruent to  $\angle DBA$ .  $\overline{AB} \cong \overline{BA}$  because they're the same segment.  $\triangle ABE \cong \triangle BAD$  because AAS.  $\therefore \overline{AE} \cong \overline{BD}$  because CPCTC

IWBAT show that pairs of angles or pairs of sides are congruent by identifying related triangles and proving them congruent, then applying CPCTC.

## 4.6 Corresponding parts of congruent triangles

Exercises p. 231 #1-5, 7-9

IWBAT show that pairs of angles or pairs of sides are congruent by identifying related triangles and proving them congruent, then applying CPCTC.

Warmup: p. 228 #15 to be turned in



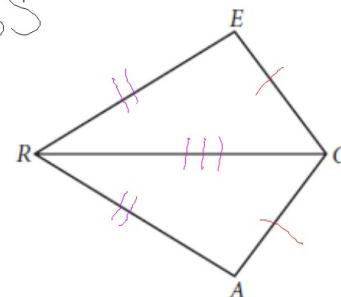
IWBAT create flowchart proofs.

I will do this via capturing notes in Cornell notes format, whole class discussions to introduce and clarify key concepts, and completing practice problems collaboratively with teacher support.

#### 4.7 Flowchart Thinking

In the figure at right,  $\overline{EC} \cong \overline{AC}$  and  $\overline{ER} \cong \overline{AR}$ . Is  $\angle A \cong \angle E$ ? If so, give a logical argument to explain why they are congruent.

SSS



$$\overline{EC} \cong \overline{AC}$$

Given

$$\overline{ER} \cong \overline{AR}$$

Given

$$\overline{RC} \cong \overline{RC}$$

Reflexive property

$$\triangle CER \cong \triangle CAR$$

SSS

CPCTC

$$\angle E \cong \angle A$$

CPCTC

IWBAT create flowchart proofs.

#### 4.7 Flowchart Thinking

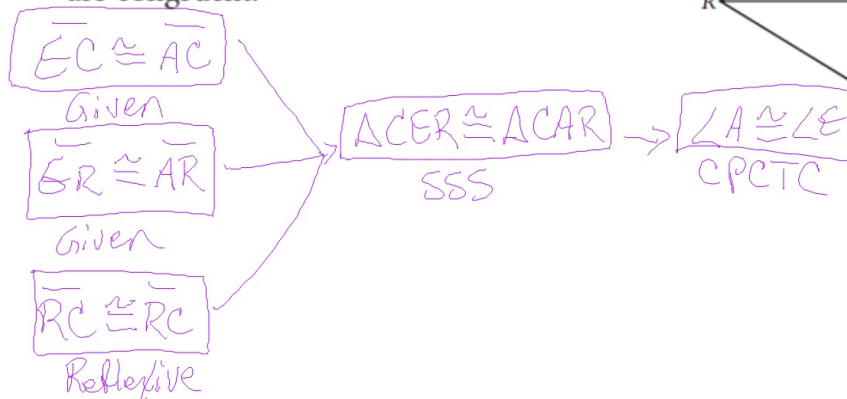
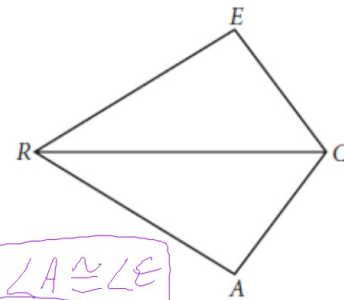
A flowchart is a concept map that shows all the steps in a complicated procedure in proper order. Arrows connect the boxes to show how facts lead to conclusions.

Flowcharts make your logic visible so that others can follow your reasoning. To present your reasoning in flowchart form, create a **flowchart proof**. Place each statement in a box. Write the logical reason for each statement beneath its box.

IWBAT create flowchart proofs.

#### 4.7 Flowchart Thinking

In the figure at right,  $\overline{EC} \cong \overline{AC}$  and  $\overline{ER} \cong \overline{AR}$ . Is  $\angle A \cong \angle E$ ? If so, give a logical argument to explain why they are congruent.



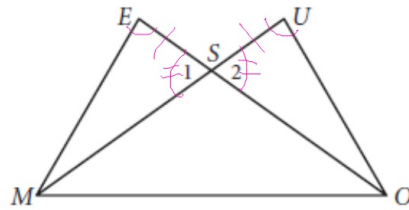
IWBAT create flowchart proofs.

## 4.7 Flowchart Thinking

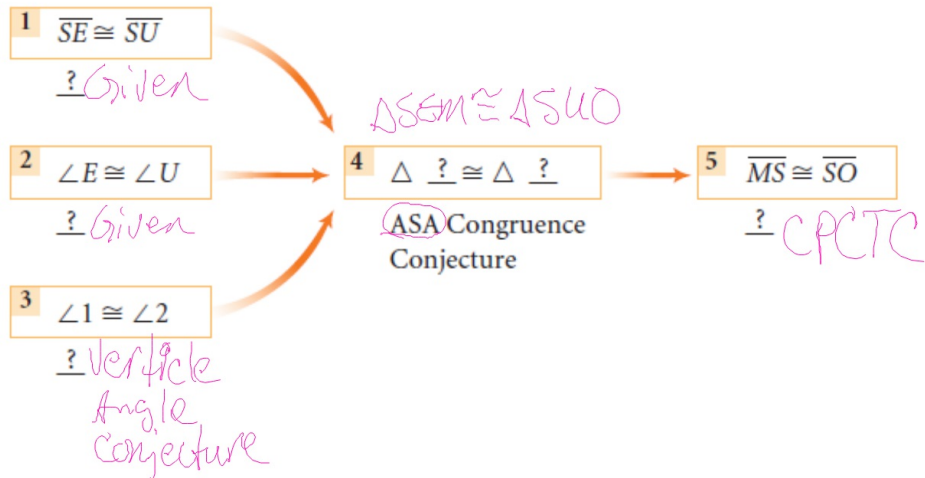
2. Copy the flowchart. Provide each missing reason or statement in the proof.

Given:  $\overline{SE} \cong \overline{SU}$   
 $\angle E \cong \angle U$

Show:  $\overline{MS} \cong \overline{OS}$



### Flowchart Proof



IWBAT create flowchart proofs.

## 4.7 Flowchart Thinking

Exercises pp. 237-238 #3, 4, 5, 7, 10

IWBAT create flowchart proofs.

Warmup: p. 231 #6 to be turned in

#### 4.8 Proving Isosceles Triangle Conjectures

02/08/18

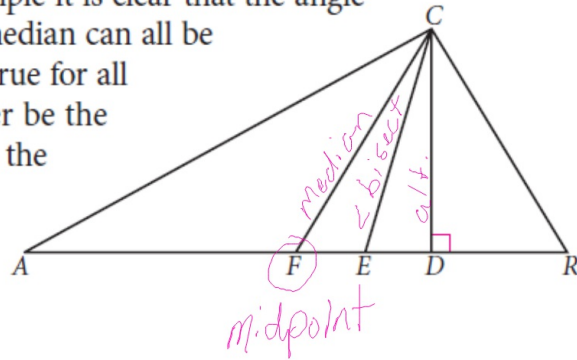
IWBAT investigate vertex angle bisectors of isosceles triangles and practice writing flowchart proofs.

I will do this via capturing notes in Cornell notes format, whole class discussions to introduce and clarify key concepts, and completing practice problems collaboratively with teacher support.



## 4.8 Proving Isosceles Triangle Conjectures

First, consider a scalene triangle. In  $\triangle ARC$ ,  $\overline{CD}$  is the altitude to the base  $\overline{AR}$ ,  $\overline{CE}$  is the angle bisector of  $\angle ACR$ , and  $\overline{CF}$  is the median to the base  $\overline{AR}$ . From this example it is clear that the angle bisector, the altitude, and the median can all be different line segments. Is this true for all triangles? Can two of these ever be the same segment? Can they all be the same segment?



IWBAT investigate vertex angle bisectors of isosceles triangles and practice writing flowchart proofs.

## 4.8 Proving Isosceles Triangle Conjectures

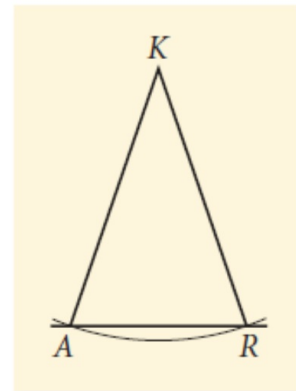
Construct a large isosceles triangle on a sheet of paper. Label it  $ARK$ , with  $K$  the vertex angle.

Construct angle bisector  $\overline{KD}$  with point  $D$  on  $\overline{AR}$ . Do  $\triangle ADK$  and  $\triangle RDK$  look congruent? If they are congruent, then  $\overline{KD}$  is a line of symmetry.

With your compass, compare  $\overline{AD}$  and  $\overline{RD}$ . Is  $D$  the midpoint of  $\overline{AR}$ ? If  $D$  is the midpoint, then what type of special segment is  $\overline{KD}$ ?

Compare  $\angle ADK$  and  $\angle RDK$ . Do they have equal measures? Are they supplementary? What conclusion can you make?

Compare your conjectures with the results of other students. Now combine the two conjectures from Steps 3 and 4 into one.



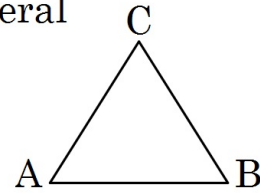
IWBAT investigate vertex angle bisectors of isosceles triangles and practice writing flowchart proofs.

## 4.8 Proving Isosceles Triangle Conjectures

### Vertex Angle Bisector Conjecture

In an isosceles triangle, the bisector of the vertex angle is also altitude and median.

Prove that this conjecture also applies to equilateral triangles by constructing your own equilateral triangle labeled ABC.



Write up your results as a paragraph proof.

Compare your proof with at least one partner at your table and discuss your results.

Working with a partner, convert your paragraph proof to a flowchart proof.

IWBAT investigate vertex angle bisectors of isosceles triangles and practice writing flowchart proofs.

## 4.8 Proving Isosceles Triangle Conjectures

### Equilateral/Equiangular Triangle Conjecture

Every equilateral triangle is equiangular, and, conversely, every equiangular triangle is equilateral.

The *Equilateral/Equiangular Triangle Conjecture* is a biconditional conjecture: Both the statement and its converse are true. A triangle is equilateral if and only if it is equiangular. One condition cannot be true unless the other is also true.

IWBAT investigate vertex angle bisectors of isosceles triangles and practice writing flowchart proofs.

## 4.8 Proving Isosceles Triangle Conjectures

Exercises pp.243-244 #1-5

IWBAT investigate vertex angle bisectors of isosceles triangles and practice writing flowchart proofs.

Warmup: p. 231 #6 revise and turn back in

Exercises DG pp. 200-201 #1-22 Evens, #9

Exercises DG pp. 206-207 #1-8, 10, 11

Exercises p.212 #4-8, 11

Exercises pp.216-217 #2-16 evens

Exercises pp.222-223 #1-5 odd, 9-15 odd

Exercises pp.227-228 #2-14 evens

Exercises p. 231 #1-5, 7-9

Exercises pp. 237-238 #3, 4, 5, 7, 10

Exercises pp.243-244 #1-5

Warmup: p. 238 #6 to be turned in



PP. 249-252 #2, 6, 8, 20, 22, 25-30, 32, 33-35

Please put the problems marked in red on a separate piece of paper to be turned in.

Complete the Unit Test